

Praktische Informatik I

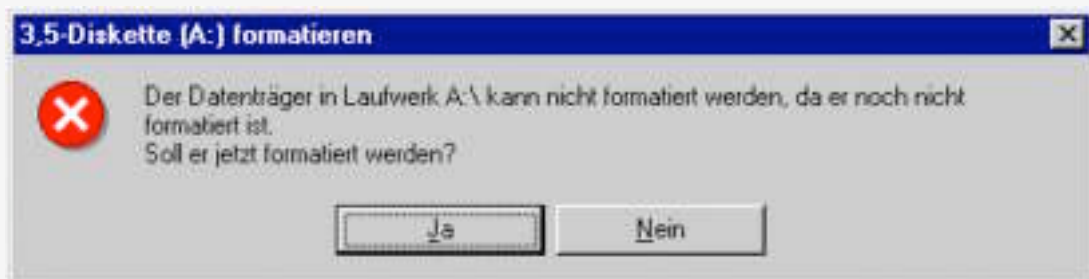
WS 2004/2005

27

waste.informatik.hu-berlin.de/Lehre

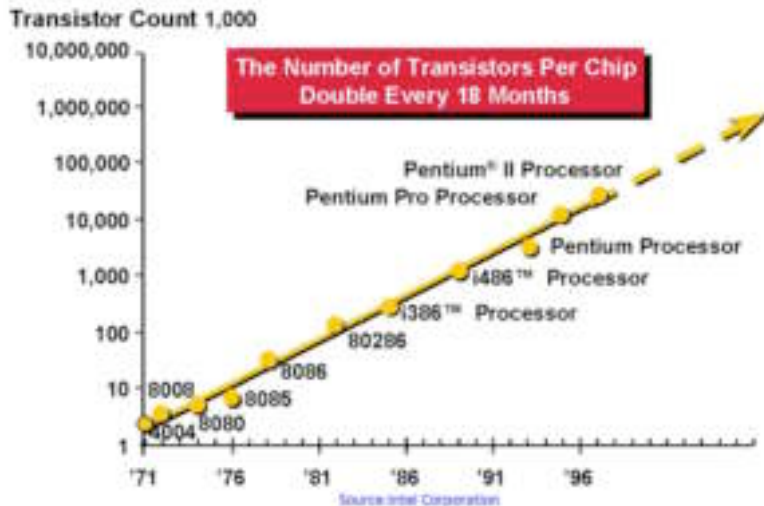


2



Wachstum der Komplexität & das Problem der Skalierbarkeit

3



Moore's Law

"The complexity for minimum component costs has increased at a rate of roughly a factor of two per year. Certainly over the short term this rate can be expected to continue, if not to increase. Over the longer term, the rate of increase is a bit more uncertain, although there is no reason to believe it will remain nearly constant for at least 10 years."

(Gordon Moore, *Electronics magazine*, April 1965)

Wachstum der Komplexität & das Problem der Skalierbarkeit

4



1975	8080	4500
1978	8086	29000
1982	80286	90000
1985	80386	229000
1989	80486	1,2 Mio
1993	pentium	3,1 Mio
1995	pentium <i>pro</i>	5,5 Mio
1998	pentium III	28 Mio
2000	pentium IV	42 Mio
2004		500 Mio

Prozessorchipfehler

5

386 A1 step	28	
386 B0 step	12	
386 B1 step	15	
386 D0 step	3	
386 "some versions"	19	
386 "all versions"	1	
Summe Intel 386		78
486 "early versions"	6	
486 "some versions"	8	
486 A-B4 step	3	
486 A-C0 step	2	
486 "all versions"	2	
Summe Intel 486		21
Pentium 60- und 66-MHz	21	
Pentium 75-, 90- und 100-MHz	42	
Summe Intel Pentium bis 1995		56

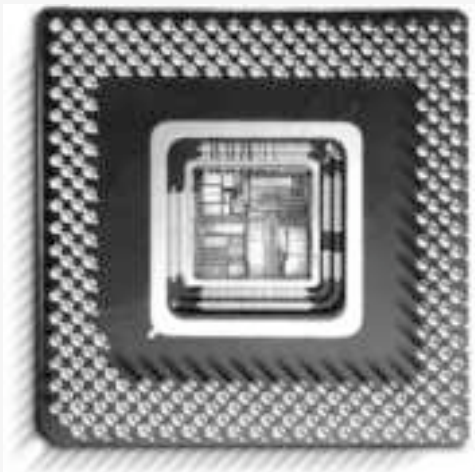


Der Pentium Fehler

6

The New York Times
ON THE WEB

November 1994



$$4195835/3145727=1,33373906 ??$$

oder

$$4195835/3145727=1.3338204491362$$

- Schaden 450 Mio. US-\$



Der Pentium Fehler

7



- ›Because we had been marketing the Pentium brand heavily, there was a bigger brand awareness,‹ says Richard Dracott, Intel director of marketing. ›We didn't realize how many people would know about it, and some people were outraged when we said it was no big deal.‹
- Intel eventually offered to replace the affected chips, which Dracott says cost the company \$450 million.
- To prove that it had learned from its mistake, Intel then started publishing a list of known ›errata,‹ or bugs, for all of its chips.

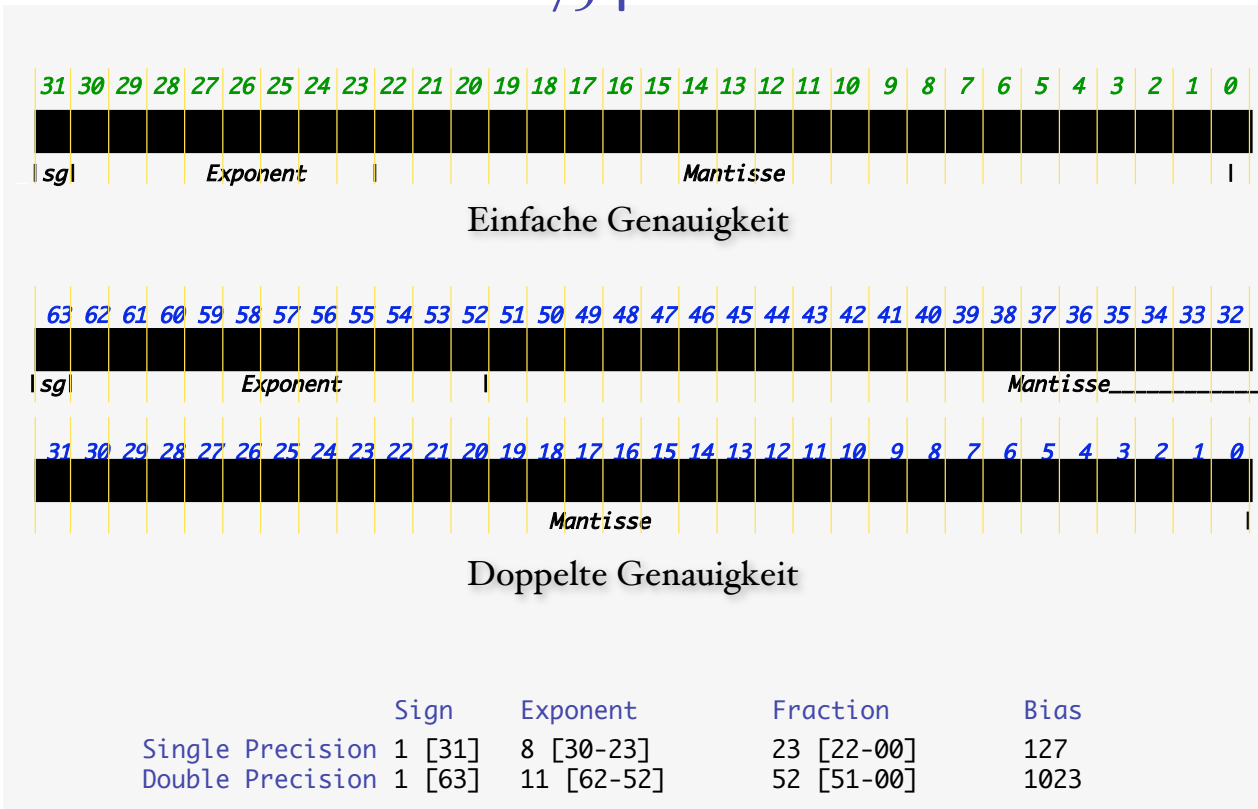
Software: Gleitkomma-Zahlen IEEE 754 Standard

8

- **Es gibt 32-bit und 64-bit Darstellungen.**
- $\langle \text{Gleitkommazahl} \rangle := \langle \text{Mantisse} \rangle \langle \text{Exponent} \rangle$
- $\langle \text{Mantisse} \rangle :=$
 $\langle \text{Dezimalzahl} \rangle |$
 $\langle \text{Dezimalzahl} \rangle . \langle \text{Dezimalzahl} \rangle |$
 $\langle \text{Dezimalzahl} \rangle . \langle \text{Dezimalzahl} \rangle$
- $\langle \text{Exponent} \rangle :=$
 $e \langle \text{Ganzzahl} \rangle |$
 $E \langle \text{Ganzzahl} \rangle$
- **float x umfaßt den Bereich**
von gerundet $1,5 \cdot 10^{-39} < x < 1,7 \cdot 10^{38}$
- **double x umfaßt den Bereich**
von gerundet $6 \cdot 10^{-309} < x < 8 \cdot 10^{307}$

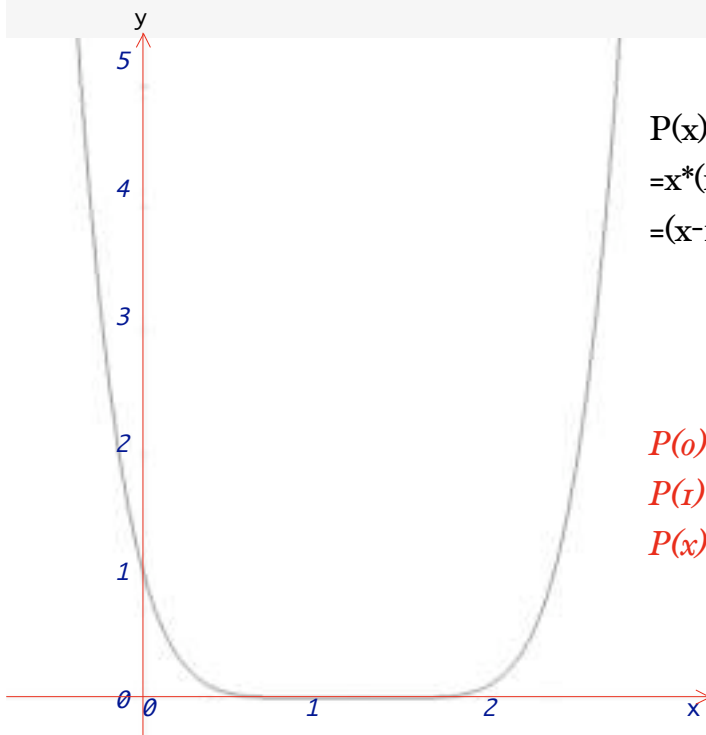
Software: Gleitkomma-Zahlen IEEE 754 Standard

9



Ein Polynom sechsten Grades $P(x)$

10



$$\begin{aligned}
 P(x) &= x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1 \\
 &= x \cdot (x \cdot (x \cdot (x \cdot (x \cdot (x - 6) + 15) - 20) + 15) - 6) + 1 \\
 &= (x - 1)^6
 \end{aligned}$$

$$P(0) = 1$$

$$P(1) = 1 - 6 + 15 - 20 + 15 - 6 + 1 = 0$$

$P(x)$ wird niemals negativ.

```

public class polynomial {
    static float Polynom(float x) {
        float x2,x3,x4,x5,x6,P;
        x2=x*x;
        x3=x2*x;
        x4=x3*x;
        x5=x4*x;
        x6=x5*x;
        P=x6-6*x5+15*x4-20*x3+15*x2-6*x+1;
        return P;}

    public static void main(String args[ ]) {
        int i,n=54;float delta,x;
        x=0.5f; delta=1/(float)n;
        for (i=0;i<=n;i++) {
            System.out.println("P(" + x + ")="+ Polynom(x));
            x=x+delta;}
        }
}

```

```

P(0.5)=0.015625
P(0.5185185)=0.012458801
P(0.537037)=0.009846449
P(0.5555555)=0.007707596
P(0.57407403)=0.0059702396
P(0.59259254)=0.0045723915
P(0.61111104)=0.0034589767
P(0.62962955)=0.0025815964
P(0.64814806)=0.001897335
P(0.66666657)=0.0013718605
P(0.6851851)=9.7322464E-4
P(0.7037036)=6.766319E-4
P(0.7222221)=4.6014786E-4
P(0.7407406)=3.0374527E-4
P(0.7592591)=1.9550323E-4
P(0.7777776)=1.206398E-4
P(0.7962961)=7.05719E-5
P(0.8148146)=4.005432E-5
P(0.83333313)=2.1457672E-5
P(0.85185164)=1.1444092E-5
P(0.87037015)=3.81739E-6
P(0.88888866)=1.9073486E-6
P(0.90740716)=0.0
P(0.9259257)=4.7683716E-7
P(0.9444442)=9.536743E-7
P(0.9629627)=-1.4305115E-6
P(0.9814812)=-9.536743E-7

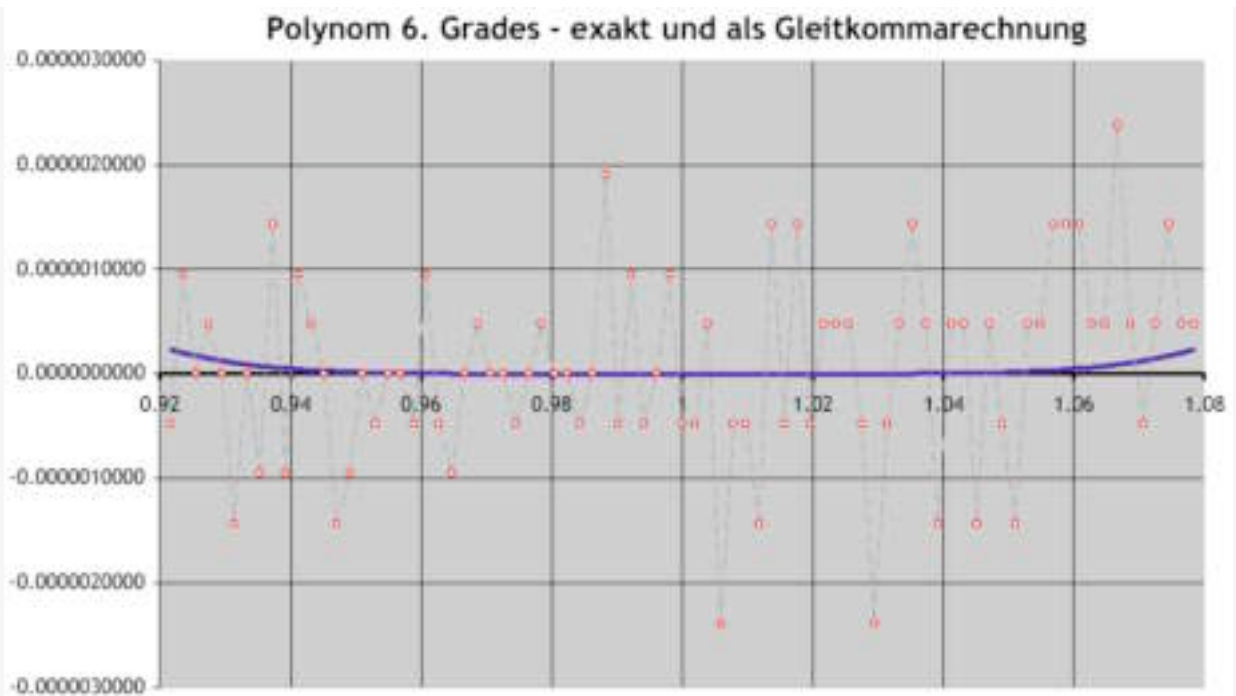
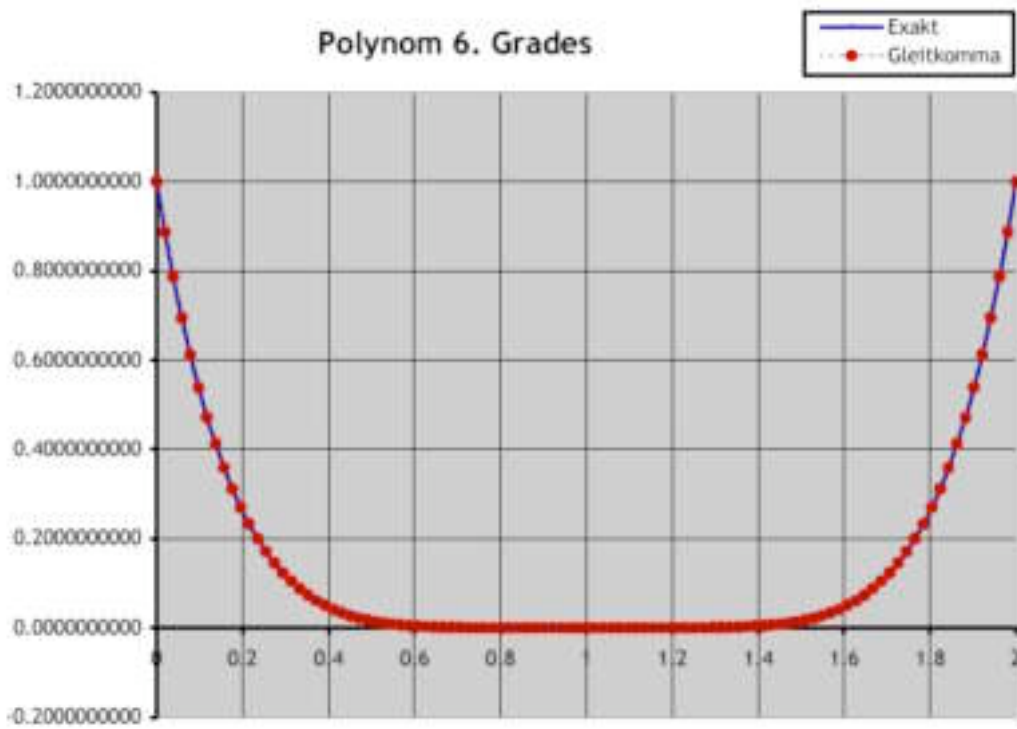
```

```

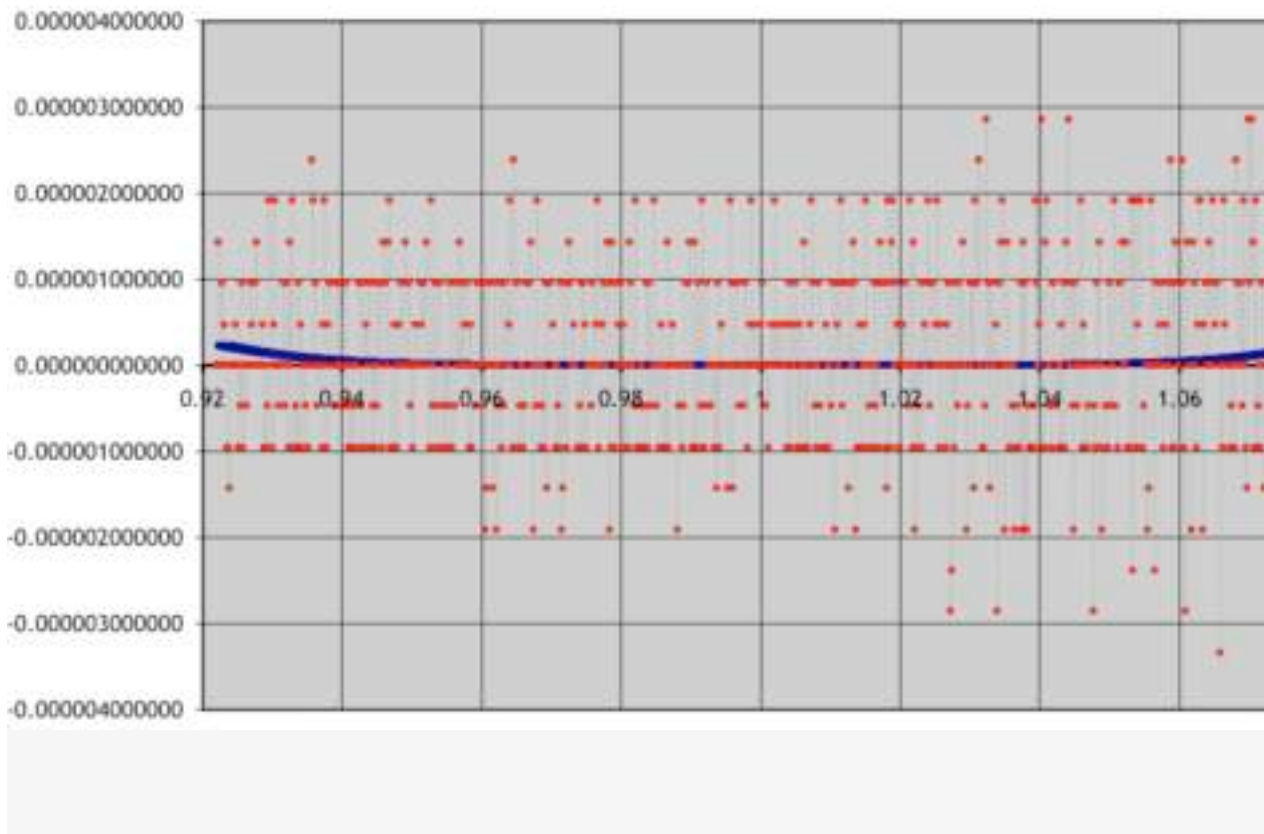
P(0.9999997)=-9.536743E-7
P(1.0185182)=-1.4305115E-6
P(1.0370368)=9.536743E-7
P(1.0555553)=-9.536743E-7
P(1.0740739)=-9.536743E-7
P(1.0925925)=2.3841858E-6
P(1.1111111)=9.536743E-7
P(1.1296296)=2.861023E-6
P(1.1481482)=9.536743E-6
P(1.1666667)=2.4318695E-5
P(1.1851853)=4.1007996E-5
P(1.2037039)=7.43866E-5
P(1.2222224)=1.2111664E-4
P(1.240741)=1.9693375E-4
P(1.2592596)=3.0326843E-4
P(1.2777781)=4.606247E-4
P(1.2962967)=6.7329407E-4
P(1.3148153)=9.7227097E-4
P(1.3333338)=0.0013742447
P(1.3518524)=0.0018959045
P(1.370371)=0.0025815964
P(1.3888895)=0.0034561157
P(1.4074081)=0.004573822
P(1.4259267)=0.00596714
P(1.4444453)=0.007709503
P(1.4629638)=0.009849548
P(1.4814824)=0.012465477
P(1.500001)=0.015623093

```

P(x) wird niemals negativ



Polynom 6. Grades - exakt und als Gleitkommarechnung



Rundungsfehler

16

$1/2 + 1/2 = 1$
 $1/3 + 1/3 + 1/3 = 1$
 $1/4 + 1/4 + 1/4 + 1/4 = 1$
 $1/5 + 1/5 + 1/5 + 1/5 + 1/5 = 1$

...

$0,5 + 0,5 = 1$
 $0,25 + 0,25 + 0,25 + 0,25 = 1$
 $0,2 + 0,2 + 0,2 + 0,2 + 0,2 = 1$

...

jedoch

$0.3 + 0.3 + 0.3 = 0.9$
 $0.33 + 0.33 + 0.33 = 0.99$
 $0.333 + 0.333 + 0.333 = 0.999$

...

Brüche zur Basis 2

$1/2 = 0,1$
 $1/3 = 0,10101010101010...$
 $1/4 = 0,01$
 $1/5 = 0,0011001100110011...$
 $1/6 = 0,0010101010101010...$
 $1/7 = 0,0010010010010010...$
 $1/8 = 0,001$
 $1/9 = 0,0001110001110001...$
 $1/10 = 0,0001100110011001...$
 $1/11 = 0,0001011101000101...$
 $1/12 = 0,0001010101010101...$
 $1/13 = 0,0001001110110001...$
 $1/14 = 0,0001001001001001...$
 $1/15 = 0,0001000100010001...$
 $1/16 = 0,0001$

...

bc

```

scale=32;
ibase=obase=2
n=1000;
h=1/n
for(i=1;i<=n;i++){
h=(n+1)*h-1
print h,"\n"
}

```

$$\begin{cases} h(0) = 1/n \\ h(i+1) = (n+1) * h(i) - 1 \end{cases}$$

Beispiel n=5

$$\begin{aligned} h(0) &= 1/5; \\ h(1) &= 6 * h(0) - 1 = 6/5 - 1 = 1/5 \\ h(2) &= 6 * h(1) - 1 = 1/5 \\ h(3) &= 6 * h(2) - 1 = 1/5 \\ h(4) &= 6 * h(3) - 1 = 1/5 \\ h(5) &= 6 * h(4) - 1 = 1/5 \end{aligned}$$

$$\dots \\ n * h(5) = 1 \quad (\text{oder allgemein } n * h(i) = 1)$$

$$\begin{cases} h(0) = 1/n \\ h(i+1) = (n+1) * h(i) - 1 \end{cases}$$

n=8 als Dezimalbruch

$$\begin{aligned} h(0) &= 0,125; \\ h(1) &= 9 * h(0) - 1 = 0,125 \\ h(2) &= 9 * h(1) - 1 = 0,125 \\ h(3) &= 9 * h(2) - 1 = 0,125 \\ h(4) &= 9 * h(3) - 1 = 0,125 \\ h(5) &= 9 * h(4) - 1 = 0,125 \\ h(6) &= 9 * h(5) - 1 = 0,125 \\ h(7) &= 9 * h(6) - 1 = 0,125 \\ h(8) &= 9 * h(7) - 1 = 0,125 \\ \dots \\ n * h(8) &= 1 \end{aligned}$$

n=8 als Binärbruch

$$\begin{aligned} h(0) &= 0,001; \\ h(1) &= 1000 * h(0) - 1 = 0,001 \\ h(2) &= 1000 * h(1) - 1 = 0,001 \\ h(3) &= 1000 * h(2) - 1 = 0,001 \\ h(4) &= 1000 * h(3) - 1 = 0,001 \\ h(5) &= 1000 * h(4) - 1 = 0,001 \\ h(6) &= 1000 * h(5) - 1 = 0,001 \\ h(7) &= 1000 * h(6) - 1 = 0,001 \\ h(8) &= 1000 * h(7) - 1 = 0,001 \\ \dots \\ n * h(8) &= 1 \end{aligned}$$

Java

```
static float Funktion(int n) {
    float h,F; long i;
    h=1/(float)n;
    /*Schleifeninvariante h=1/n */
    for (i=1;i<=n;i++){h=(float)(n+1)*h-1;}
    /*Schleifeninvariante h=1/n */
    F=n*h;
    return F;
    /* F=n*1/n, also F=1 */
}
```

$$\begin{cases} h(0) = 1/n \\ h(i+1) = (n+1) * h(i) - 1 \end{cases}$$

n=1, f(n)=1.0
 n=2, f(n)=1.0
 n=3, f(n)=1.0000019
 n=4, f(n)=1.0
 n=5, f(n)=1.000309
 n=6, f(n)=1.0080142
 n=7, f(n)=1.09375
 n=8, f(n)=1.0
 n=9, f(n)=48.683716
 n=10, f(n)=563.1785
 n=11, f(n)=22144.375
 n=12, f(n)=854569.75
 n=13, f(n)=4.0550648E7
 n=14, f(n)=2.32004582E9
 n=15, f(n)=6.0129542E10
 n=16, f(n)=1.0

2,3 * 10⁹

$$\begin{cases} h(0) = 1/n \\ h(i+1) = (n+1) * h(i) - 1 \end{cases}$$

```

class hFunktion {
static float Funktion(int n) {
float h,F; long i;
h=1/(float)n;
/*Schleifeninvariante h=1/n */
for
(i=1;i<=n;i++){h=(float)(n+1)*h-1;}
/*Schleifeninvariante h=1/n */
F=n*h;
return F;
/* F=n*1/n, also F=1 */
}

public static void main(
String[] args ) {
int i=1;
for (i=1;i<=34;i++){
System.out.println("n="+i +",
f(n)="+ Funktion(i));
} //Ende for
} //Ende main
} //Ende class
  
```

n=17, f(n)=1.15813768E14
 n=18, f(n)=2.61325164E15
 n=19, f(n)=1.87499976E17
 n=20, f(n)=1.895213E19
 n=21, f(n)=8.4093746E20
 n=22, f(n)=6.5875305E22
 n=23, f(n)=-5.5206144E23
 n=24, f(n)=2.7105054E26
 n=25, f(n)=-8.684812E27
 n=26, f(n)=4.3506622E29
 n=27, f(n)=-2.5199321E31
 n=28, f(n)=8.7350875E33
 n=29, f(n)=-2.7271266E35
 n=30, f(n)=4.65824E37
 n=31, f(n)=-Infinity
 n=32, f(n)=1.0
 n=33, f(n)=-Infinity
 n=34, f(n)=Infinity

$f(n) \in \{-\infty, +\infty\}$

$$\begin{cases} h(0) = 1/n \\ h(i+1) = (n+1) * h(i) - 1 \end{cases}$$

```

static float Funktion(int n) {
float h,F; long i;
h=1/(float)n;
/*Schleifeninvariante h=1/n */
for
(i=1;i<=n;i++){h=(float)(n+1)*h-1;}
/*Schleifeninvariante h=1/n */
F=n*h;
return F;
/* F=n*1/n, also F=1 */
}

public static void main(
String[] args ) {
int i=1;
for (i=1;i<=34;i++){
System.out.println("n="+i +",
f(n)="+ Funktion(i));
} //Ende for
} //Ende main
} //Ende class
  
```


$$\begin{cases} h(0) = 1/n \\ h(i+1) = (n+1) * h(i) - 1 \end{cases}$$

n=5 als Dezimalbruch

$$\begin{aligned} h(0) &= 0,2; \\ h(1) &= 6 * h(0) - 1 = 0,2 \\ h(2) &= 6 * h(1) - 1 = 0,2 \\ h(3) &= 6 * h(2) - 1 = 0,2 \\ h(4) &= 6 * h(3) - 1 = 0,2 \\ h(5) &= 6 * h(4) - 1 = 0,2 \end{aligned}$$

$$\dots \\ n * h(5) = 1$$

n=5 als Binärbruch

$$\begin{aligned} h(0) &= 0,00110011; \\ h(1) &= 110 * h(0) - 1 = 0,00110010 \\ h(2) &= 110 * h(1) - 1 = 0,00101100 \\ h(3) &= 110 * h(2) - 1 = 0,00001000 \\ h(4) &= 110 * h(3) - 1 = -0,11010000 \\ h(5) &= 110 * h(4) - 1 = -101,111000 \end{aligned}$$

$$\dots \\ n * h(5) = -11101.01100 (=29,375)$$

scale=1000

ibase=2

obase=2

n=1111;

h=1/n

for(i=1; i<=n; i++){h=(n+1)*h-1

print h, "\t", i, "\n"

}

.000100010001000100001111010	1
.000100010001000011110100011	10
.0001000100001111101000110111	11
.0001000011111010001101110111	100
.0000111110100011011101111110	101
-.0000101111001000100000010110	110
-1.101110010001000000101100011	111
-11100.1001000100000010110001111001	1000
-111001010.000100000010110001110010111	1001
-1110010100010.000000101100011100101111101	1010
-11100101000100001.001011000111001011111010100	1011
-111001010001000010011.110001110010111110101001111	1100
-1110010100010000100111101.011100101111101010011110011	1101
-111001010001000010011110111000.001011111010100111100110110	1110
-1110010100010000100111101110000011.111110101001111001101100001	1111

Intervall-Arithmetik

$$X = X_{\text{center}} \pm \varepsilon$$

oder mengentheoretisch

$$x \in \{ [\min\{x\} \dots \max\{x\}] \mid x \in \mathbb{Q}, \min\{x\} \in \mathbb{Q}, \max\{x\} \in \mathbb{Q}, \min\{x\} \leq \max\{x\} \}$$

z.B.

$$x \in [0,992187 \dots 0,995312]$$

$$x \in [0 \dots 1]$$

$$x \in [100 \dots 190]$$

Intervallarithmetische Regeln

- $[a,b] + [c,d] = [a+c, b+d]$
- $[a,b] - [c,d] = [a-d, b-c]$
- $[a,b] * [c,d] = [\min\{a*c, a*d, b*c, b*d\}, \max\{a*c, a*d, b*c, b*d\}]$
- $[a,b] / [c,d] = [a,b] * [1/c, 1/d]$, sofern $0 \notin [c,d]$
- $[a,b] + [c,d] = [c,d] + [a,b]$

Kommutativität

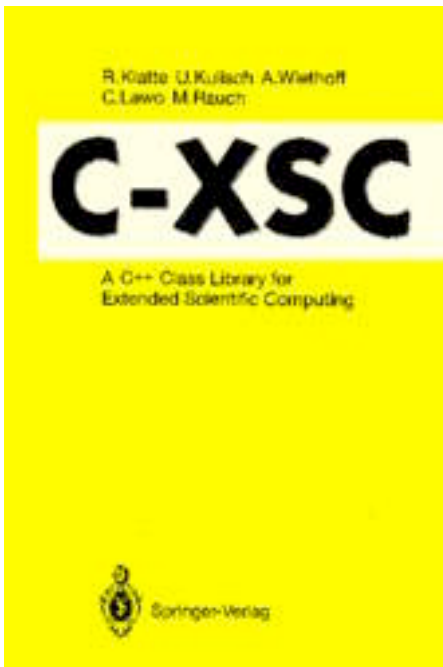
- $[a,b] * [c,d] = [c,d] * [a,b]$
- $[a,b] + ([c,d] + [e,f]) = ([a,b] + [c,d]) + [e,f] = [a,b] + [c,d] + [e,f]$

Assoziativität

- $[a,b] * ([c,d] * [e,f]) = ([a,b] * [c,d]) * [e,f] = [a,b] * [c,d] * [e,f]$

Neutrale Elemente

- $[a,b] * 1 = [a,b]$
- $[a,b] + 0 = [a,b]$



Es gilt nicht:

Lösbarkeiten

$$[a,b] + [c,d] = 0$$

$$[a,b] * [c,d] = 1$$

Beide Gleichungen sind nicht eindeutig lösbar für $a < b, c < d$

Im allgemeinen, gilt nicht:

$$[a,b] - [a,b] = [0,0]$$

$$[a,b] / [a,b] = [1,1]$$

Statt des Distributivgesetzes gilt ein Subdistributivgesetz:

$$[a,b] * ([c,d] + [e,f]) \subseteq ([a,b] * [c,d]) + ([a,b] * [e,f])$$

Pascal SC, Fortran SC, Calculus, Acrith-XSC, C-XSC

Erweiterung von Fortran, Pascal oder C durch Intervallarithmetik mit 29 intervallarithmetische Operatoren wie z.B.

$+, -, *, /, \text{div}, \text{mod}, \text{and}, \text{or}, \dots$

sowie

$<, >, \leq, \geq, \dots$

und intervallarithmetische Grundfunktionen wie

sin, cos, exp, ln, arctan, sqrt

Hinzu kommen spezielle Intervalloperatoren z.B. für die Matrixmultiplikation

$a * x + b$

The image shows a screenshot of the website <http://www.math.uni-wuppertal.de/~xsc/>. The website is titled "Scientific Computing / Software Engineering" and "Faculty C / Department of Mathematics, University of Wuppertal". The main heading is "XSC Languages (C-XSC, PASCAL-XSC)" with the subtitle "Scientific Computing with Validation, Arithmetic Requirements, Hardware Solution and Language Support". A table provides links to various versions and software:

C-XSC 2.0	Pascal-XSC (binary version)	Pascal-XSC BCD (decimal version)
About	About	About
Documentation	Documentation	Documentation
Free Download (C++ Library and Toolbox)	Free Download (Compiler and Toolbox)	Free Download (Compiler)
Additional Software	Additional Software	not yet available
Archive Older Versions (C-XSC 1.x)	Older Tools	not yet available
History of XSC-Languages and Credits		

Below the table, it says "(Links to other interval software of the WRSWT-Group: [FiLib++](#), [intpakX](#))". To the left of the website screenshot is the cover of the book "C++ Toolbox for Verified Computing: Basic Numerical Problems" by R. Harzner, M. Hecker, U. Kulisch, and D. Rätz, published by Springer-Verlag.

<http://www.math.uni-wuppertal.de/~xsc/>

SC Geschichte (Scientific Computing)

34

Institute of Applied Mathematics (Prof. U. Kulisch), University of Karlsruhe:

- 1967: An ALGOL-60 extension implemented on a Zuse Z 23 computer with operators and a number of elementary functions for a new data type interval.
- 1968/69: Implementation of the above language on a more powerful computer, an Electrologica X8.
- 1976: PASCAL-SC, a PASCAL extension implemented on a Z-80 microprocessor with 64 KB main memory (funded by the German company Nixdorf). The programming convenience of PASCAL-SC allowed a small group of collaborators to implement a large number of problem solving routines with automatic result verification within a few months.
- 1980: PASCAL-SC with a large number of problem solving routines was exhibited at the Hannover fair.
- 1980/89: ACRITH-XSC, a FORTRAN 77 extension for the /370 architecture was developed and implemented in cooperation with IBM.
- 1983: IBM shipped the first edition of the ACRITH library.
- 1986: ARITHMOS for BS 2000 (with Siemens)
- 1990: IBM shipped ACRITH-XSC
- 1990/91: Development of a new Runtime System (RTS) for PASCAL-XSC in C
- 1991: PASCAL-XSC shipped. The PASCAL-XSC system compiles a given PASCAL-XSC source code into C code which is passed to a C-Compiler.
- 1992: C++ class library C-XSC shipped, available for many computers with C++ compiler translating der AT&T language standard 2.0
- 1993: "Numerical Toolbox for Verified Computing" in PASCAL-XSC is published by Springer-Verlag.
- 1994: "C++ Toolbox for Verified Computing" in C-XSC is published by Springer-Verlag
- 1996: Begin of the implementation of Fortran-XSC (TU Dresden, Prof. Walter)
- 1997/98: Oberon-XSC (Universität Karlsruhe, Dr. P. Januschke; ETH Zürich, Prof. Gutknecht)
- 1997: XSC General Public License, *all XSC software has been available FREE OF CHARGE since 1997.*

WRSWT-Group (Prof. W. Krämer), University of Wuppertal:

- 1999/2001: Redesign of C-XSC (collaboration of the Institute of Applied Mathematics (Prof. Kulisch), University of Karlsruhe and the WRSWT-Group (Prof. Krämer), University of Wuppertal)
- 2000/02: New Web-Presentation of XSC-Languages and Additional Software
- 2002: First Beta Release C-XSC 2.0

Bewiesenermaßen sichere Hardware?³⁵

Das VIPER-Projekt

Verifiable Integrated Processor for Enhanced Reliability

Avra J. Cohn. A proof of correctness of the VIPER microprocessor: The first level. In Graham Birtwistle and P.A. Subrahmanyam, editors, *VLSI Specification, Verification and Synthesis*, pages 27–71. Kluwer Academic Publishers, 1987.

Das VIPER-Projekt -

36

verifiable integrated processor for enhanced reliability

- VIPER is a 32-bit microprocessor architecture designed by the Royal Signals and Radar Establishment (RSRE) in Malvern, England.

Logische Verifikation eines Mikroprozessors 1995

- **The reader may be surprised that the majority of formally verified microprocessors have been part of University projects, but this is exactly the problem with hardware verification:**
- The industry is reluctant to adapt these methods. ...

Das VIPER-Projekt - *verifiable integrated processor for enhanced reliability*

- **Various difficulties arise at this point. Firstly, because the theorem provers use a different language than the usual description language errors may occur in translating into the formal language. Secondly, most of the current theorem provers/checkers are complex to use, slow and do not have a standard language.** Engineers require a strong mathematical background to use them. ...
- **In spite of new opportunities offered by hardware verification in industry, there are limitations of formal proof in hardware verification.**
- **Firstly, a proof does not show that the circuit will behave as intended by the designer, it only shows that it behaves in accordance to a possibly inaccurate specification.** Or more generally expressed, as verifiers, designers and manufacturers are all working with models, **inaccuracy in the models** can lead to errors not detectable by a formal proof.
- **Secondly, the proof does not consider extra-logical factors, such as the reset switch being pressed manually, for example. To ensure the safety of a critical system factors as staff training and performance of mechanical parts of the system have to be considered.**
- **Finally, manufacturing errors can always occur.** Therefore, one has to be careful not to neglect these factors when designing a safety critical system with a verified processor.

Murphys Law

- Was schief gehen kann, geht schief...

oder

Murphys Law

- Was schief gehen kann, geht schief...

oder

- Was nicht schief gehen kann, geht schief?

Murphys Law



Capt. Edward A. Murphy,
Air Force Project MX981

+